



Derivation of a Point-Mass Aircraft Model used for Fast-Time Simulation

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McLean, VA

Dr. Lesley A. Weitz

April 2015

Approved By:

Robert C. Strain, Department Head

Date

Elida C. Smith, Outcome Leader

Date

Abstract

This document describes a closed-loop aircraft model for testing the performance of Flight-deck Interval Management (FIM) avionics. The derivation of a point-mass aircraft model with and without winds is presented. Guidance control laws used to track a four-dimensional trajectory, described as a reference horizontal path, a vertical profile, and an indicated airspeed command, are also presented. In particular, two speed management modes, used to control the aircraft's speed during descent while also managing the aircraft's altitude relative to the reference vertical profile, are described. The data format of the reference horizontal path is described in detail, including an algorithm for mapping the aircraft's position in the horizontal plane onto the reference horizontal path. Lastly, parameters for representing different aircraft types using the general equations are enumerated.

Keywords: aircraft dynamics, aircraft control.

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Table of Contents

1	Introduction	1
2	Equations of Motion	1
2.1	Reference Frames	1
2.2	Kinematics	2
2.3	Dynamics	3
3	Equations of Motion with Winds	4
3.1	Derivation	4
3.2	Sign Convention for the Wind Gradients	6
4	Guidance Control Laws	7
4.1	Lateral Control	8
4.2	Speed Control Using Thrust	9
4.3	Speed Control Using Pitch	11
4.4	Control Gains and Parameter Values	13
5	Reference Horizontal Path	14
5.1	Data Format for the Horizontal Path	14
5.2	Mapping to Along-Path Position	16
5.2.1	Cross-track Error Calculation	17
5.2.2	Along-Path Position Calculation	19
6	Aircraft-Specific Modeling	20
7	References	23
	Appendix A Abbreviations and Acronyms	24

List of Figures

1	Sequence of three simple rotations: 3-2-1 Euler angle orientation sequence.	2
2	Free-body diagram of forces acting on the point mass.	3
3	Example wind profile.	6
4	TAS and wind vectors in the horizontal plane.	8
5	Data format for the reference horizontal path.	14
6	Horizontal path generated from the data in Figure 5.	15
7	High-level process for determining the along-path position by projecting the aircraft's x, y position onto the reference horizontal path.	16
8	Illustration of quadrants around each HPT.	18
9	Projection of aircraft positions onto horizontal path to determine DTG to end point (HPT_1).	19

List of Tables

1	Control Gains and Parameter Values	13
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1 Introduction

Equations of motion (EOM) are derived for a point-mass aircraft model. The EOM may be used for modeling aircraft motion in a fast-time simulation environment. In Section 2, the EOM are derived by first defining reference frames, determining the aircraft acceleration in an appropriate reference frame, and the dynamics are derived using Newton’s Second Law. Section 3 shows the derivation of the EOM for non-zero wind conditions.

Control laws for tracking a reference four-dimensional trajectory are presented in Section 4. A detailed description of the reference horizontal path and a mapping between the aircraft’s position in the horizontal plane and the reference path is provided in Section 5.

In Section 6, an approach to representing aircraft-specific performance is presented, including a mapping to parameters from EUROCONTROL’s Base of Aircraft Data (BADA) [2].

2 Equations of Motion

2.1 Reference Frames

Assuming a flat, non-rotating Earth, an inertial reference frame N is defined with the \hat{n}_1 axis aligned with east, the \hat{n}_2 axis aligned with north, and the \hat{n}_3 axis pointing up from the Earth.

A sequence of three simple rotations relates the inertial reference frame N to a non-inertial, rotating reference frame C . Forces acting on the aircraft are most easily coordinatized in reference frame C , which is often referred to as a body-fixed reference frame when deriving the full six degree-of-freedom (DOF) EOM. The sequence of simple rotations is illustrated in Figure 1. The first rotation is through an angle ψ about the \hat{n}_3 axis (a “3” rotation) to an intermediate reference frame A ; the second rotation is through an angle γ about the \hat{a}_2 axis (a “2” rotation) to an intermediate reference frame B ; and, the third rotation is through an angle ϕ about the \hat{b}_1 axis (a “1” rotation) to reference frame C . This rotation sequence is known as a 3-2-1 Euler angle rotation sequence, which represents the orientation of the aircraft through the yaw (ψ), flight-path angle (γ), and roll (ϕ) angles.

A vector represented in the inertial frame may also be coordinatized in the C reference frame. The vector is acted upon by the rotation matrix $[C_{321}(\psi, \gamma, \phi)]$, which is the product of the three simple rotation matrices: $[C_1(\phi)][C_2(\gamma)][C_3(\psi)]$.

$$[C_{321}(\psi, \gamma, \phi)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \gamma & 0 & -\sin \gamma \\ 0 & 1 & 0 \\ \sin \gamma & 0 & \cos \gamma \end{bmatrix} \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$= \begin{bmatrix} \cos \gamma \cos \psi & \cos \gamma \sin \psi & -\sin \gamma \\ -\cos \phi \sin \psi + \sin \phi \sin \gamma \cos \psi & \cos \phi \cos \psi + \sin \phi \sin \gamma \sin \psi & \sin \phi \cos \gamma \\ \sin \phi \sin \psi + \cos \phi \sin \gamma \cos \psi & -\sin \phi \cos \psi + \cos \phi \sin \gamma \sin \psi & \cos \phi \cos \gamma \end{bmatrix} \quad (2)$$

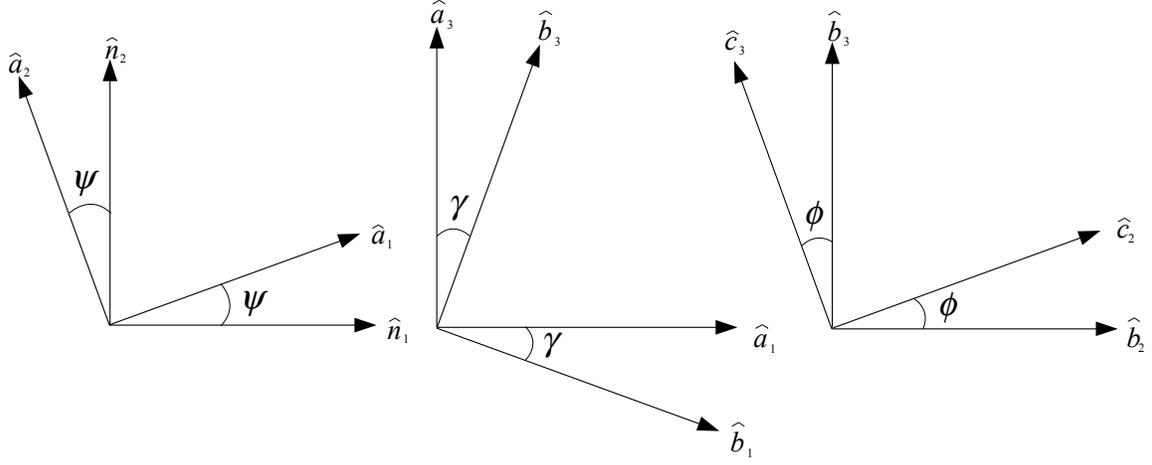


Figure 1. Sequence of three simple rotations: 3-2-1 Euler angle orientation sequence.

2.2 Kinematics

The inertial position of the aircraft is given by the following.

$$\mathbf{P} = x\hat{n}_1 + y\hat{n}_2 + h\hat{n}_3 \quad (3)$$

The inertial velocity can be written in the N and C reference frames.

$$\mathbf{V} = \dot{x}\hat{n}_1 + \dot{y}\hat{n}_2 + \dot{h}\hat{n}_3 = V\hat{c}_1 \quad (4)$$

Here, V is the aircraft's true airspeed (TAS), which is equal to the inertial velocity only when there are no winds.

To derive the EOM using Newton's Second Law, the inertial acceleration, coordinatized in the C frame, is determined using transport theorem [1].

$$\frac{{}^N d}{dt}(\mathbf{V}) = \frac{{}^C d}{dt}(\mathbf{V}) + \boldsymbol{\omega}_{C/N} \times \mathbf{V} \quad (5)$$

The angular-velocity vector relating the C frame to the N frame, $\boldsymbol{\omega}_{C/N}$, is given by the addition of angular velocities. Here, the angular velocities are expressed in the C frame by pre-multiplying by the appropriate rotation matrices.

$$\boldsymbol{\omega}_{C/N} = \boldsymbol{\omega}_{C/B} + \boldsymbol{\omega}_{B/A} + \boldsymbol{\omega}_{A/N} \quad (6)$$

$$= [\dot{\phi}, 0, 0]^T + [C_1(\phi)][0, \dot{\gamma}, 0]^T + [C_1(\phi)][C_2(\gamma)][0, 0, \dot{\psi}]^T \quad (7)$$

$$\boldsymbol{\omega}_{C/N} = (\dot{\phi} - \dot{\psi} \sin \gamma) \hat{c}_1 + (\dot{\gamma} \cos \phi + \dot{\psi} \cos \gamma \sin \phi) \hat{c}_2 + (-\dot{\gamma} \sin \phi + \dot{\psi} \cos \gamma \cos \phi) \hat{c}_3 \quad (8)$$

Making the appropriate substitutions into equation (5) yields the following expression for the inertial acceleration.

$$\dot{\mathbf{V}} = \dot{V}\hat{c}_1 + V(-\dot{\gamma} \sin \phi + \dot{\psi} \cos \gamma \cos \phi) \hat{c}_2 - V(\dot{\gamma} \cos \phi + \dot{\psi} \cos \gamma \sin \phi) \hat{c}_3 \quad (9)$$

The kinematic equations are determined by projecting $V\hat{c}_1$ into the inertial frame using the transpose of the rotation matrix, $[C_{321}(\psi, \gamma, \phi)]^T$; \dot{x} , \dot{y} , and \dot{h} are the velocities along the inertial axes, \hat{n}_1 , \hat{n}_2 , and \hat{n}_3 , respectively.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} \cos \gamma \cos \psi & -\cos \phi \sin \psi + \sin \phi \sin \gamma \cos \psi & \sin \phi \sin \psi + \cos \phi \sin \gamma \cos \psi \\ \cos \gamma \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \gamma \sin \psi & -\sin \phi \cos \psi + \cos \phi \sin \gamma \sin \psi \\ -\sin \gamma & \sin \phi \cos \gamma & \cos \phi \cos \gamma \end{bmatrix} \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} \quad (10)$$

The inertial velocity of the aircraft can be expressed as shown.

$$V = V \cos \gamma \cos \psi \hat{n}_1 + V \cos \gamma \sin \psi \hat{n}_2 - V \sin \gamma \hat{n}_3 \quad (11)$$

2.3 Dynamics

Figure 2 shows free-body diagrams of the forces acting on the point mass (the depiction of the aircraft in the figure is used for illustrative purposes only). The thrust and drag forces, T and D , respectively, are shown to be aligned with the airspeed vector in the \hat{c}_1 direction; the lift force, L , is in the \hat{c}_3 direction; and, the weight of the aircraft is in the \hat{n}_3 direction.

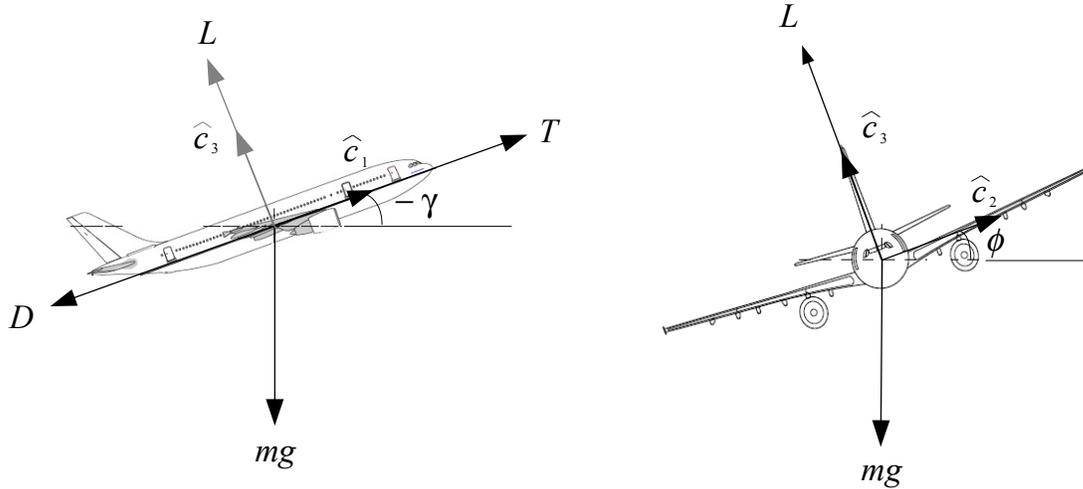


Figure 2. Free-body diagram of forces acting on the point mass: side view (left) and front view (right). The lift vector and the \hat{c}_3 unit vector are shown in gray in the side view to indicate that they are projections onto the vertical plane.

The summation of forces acting on the aircraft are given in equation (12).

$$\sum \mathbf{F} = (T - D + mg \sin \gamma) \hat{c}_1 - mg \cos \gamma \sin \phi \hat{c}_2 + (L - mg \cos \gamma \cos \phi) \hat{c}_3 \quad (12)$$

The dynamics equations, expressed in the C reference frame, are shown below.

$$m\dot{V} = T - D + mg \sin \gamma \quad (13)$$

$$mV(-\dot{\gamma} \sin \phi + \dot{\psi} \cos \gamma \cos \phi) = -mg \cos \gamma \sin \phi \quad (14)$$

$$-mV(\dot{\gamma} \cos \phi + \dot{\psi} \cos \gamma \sin \phi) = L - mg \cos \gamma \cos \phi \quad (15)$$

To decouple equations (14) and (15), equation (14) is multiplied by $\sin \phi$ and equation (15) is multiplied by $\cos \phi$ and the resulting expressions are added to get an equation for $\dot{\gamma}$.

$$mV\dot{\gamma} = -L \cos \phi + mg \cos \gamma \quad (16)$$

In a similar manner, equation (14) is multiplied by $\cos \phi$ and equation (15) is multiplied by $\sin \phi$ and the second expression is subtracted from the first to get an equation for $\dot{\psi}$.

$$mV \cos \gamma \dot{\psi} = -L \sin \phi \quad (17)$$

The aircraft EOM are the following six first-order ordinary differential equations (ODEs), comprised of three kinematic and three dynamic equations.

$$\dot{x} = V \cos \gamma \cos \psi \quad (18)$$

$$\dot{y} = V \cos \gamma \sin \psi \quad (19)$$

$$\dot{h} = -V \sin \gamma \quad (20)$$

$$\dot{V} = \frac{T - D}{m} + g \sin \gamma \quad (21)$$

$$\dot{\gamma} = -\frac{L \cos \phi}{mV} + \frac{g \cos \gamma}{V} \quad (22)$$

$$\dot{\psi} = -\frac{L \sin \phi}{mV \cos \gamma} \quad (23)$$

3 Equations of Motion with Winds

3.1 Derivation

Suppose there is a constant wind field with wind velocity expressed in the inertial reference frame.

$$\mathbf{V}_w = V_{wx} \hat{\mathbf{n}}_1 + V_{wy} \hat{\mathbf{n}}_2 \quad (24)$$

The inertial velocity of the aircraft can be expressed as

$$\mathbf{V}_{C/N} = \mathbf{V}_{C/W} + \mathbf{V}_{W/N}, \quad (25)$$

where $\mathbf{V}_{C/W}$ is the velocity of the aircraft relative to the wind frame \mathbf{W} , and $\mathbf{V}_{W/N}$ is the wind velocity relative to the inertial frame, which moves with velocity \mathbf{V}_w relative to the inertial frame. The inertial velocity is found by adding equations (11) and (24).

$$\mathbf{V} = (V \cos \gamma \cos \psi + V_{wx}) \hat{\mathbf{n}}_1 + (V \cos \gamma \sin \psi + V_{wy}) \hat{\mathbf{n}}_2 - V \sin \gamma \hat{\mathbf{n}}_3 \quad (26)$$

Given a constant wind field, the inertial velocity in equation (26) and the dynamics equations (21)-(23) comprise the EOM.

However, given a non-constant wind field, the time derivative of the wind velocity must be considered in the dynamics. The inertial acceleration, expressed in the \mathbf{C} frame, has the following form

when wind-velocity derivatives are included.

$$\begin{aligned} \dot{\mathbf{V}} = & \left[\dot{V} + (\dot{V}_{wx}c\gamma c\psi + \dot{V}_{wy}c\gamma s\psi) \right] \hat{\mathbf{e}}_1 + \\ & \left[V(-\dot{\gamma}s\phi + \dot{\psi}c\gamma c\phi) + \dot{V}_{wx}(-c\phi s\psi + s\phi s\gamma c\psi) + \dot{V}_{wy}(c\phi c\psi + s\phi s\gamma s\psi) \right] \hat{\mathbf{e}}_2 + \\ & \left[-V(\dot{\gamma}c\phi + \dot{\psi}c\gamma s\phi) + \dot{V}_{wx}(s\phi s\psi + c\phi s\gamma c\psi) + \dot{V}_{wy}(-s\phi c\psi + c\phi s\gamma s\psi) \right] \hat{\mathbf{e}}_3 \end{aligned} \quad (27)$$

Here, $s(\cdot) \equiv \sin(\cdot)$ and $c(\cdot) \equiv \cos(\cdot)$.

Applying Newton's Second Law for the inertial acceleration in equation (27) and decoupling the equations for $\dot{\gamma}$ and $\dot{\psi}$ (as described in Section 2.3) results in the following set of equations.

$$\dot{V} = \frac{T - D}{m} + g \sin \gamma - \dot{V}_{wx} \cos \gamma \cos \psi - \dot{V}_{wy} \cos \gamma \sin \psi \quad (28)$$

$$\dot{\gamma} = \frac{-L \cos \phi + mg \cos \gamma}{mV} + \frac{1}{V} (\dot{V}_{wx} \sin \gamma \cos \psi + \dot{V}_{wy} \sin \gamma \sin \psi) \quad (29)$$

$$\dot{\psi} = -\frac{L \sin \phi}{mV \cos \gamma} + \frac{1}{V \cos \gamma} (\dot{V}_{wx} \sin \psi - \dot{V}_{wy} \cos \psi) \quad (30)$$

Often wind velocities are provided in a grid format, where the wind speed and direction are given at a point in the horizontal plane and at an altitude for a given time. Assuming that V_{wx} and V_{wy} are functions of t , x , y , and h , the time derivative of V_{wx} and V_{wy} can be computed by the chain rule. For example:

$$\dot{V}_{wx}(t, x, y, h) = \frac{dV_{wx}}{dt} = \frac{\partial V_{wx}}{\partial t} + \left(\frac{\partial V_{wx}}{\partial x} \right) \left(\frac{\partial x}{\partial t} \right) + \left(\frac{\partial V_{wx}}{\partial y} \right) \left(\frac{\partial y}{\partial t} \right) + \left(\frac{\partial V_{wx}}{\partial h} \right) \left(\frac{\partial h}{\partial t} \right) \quad (31)$$

$$= \frac{\partial V_{wx}}{\partial t} + \left(\frac{\partial V_{wx}}{\partial x} \right) \dot{x} + \left(\frac{\partial V_{wx}}{\partial y} \right) \dot{y} + \left(\frac{\partial V_{wx}}{\partial h} \right) \dot{h} \quad (32)$$

Additionally, it can be assumed that the wind conditions change slowly with time ($\partial V_{wx}/\partial t \approx 0$). If the wind gradients in the x and y directions are much less than the wind gradient with respect to altitude ($\partial V_{wx}/\partial x, \partial V_{wx}/\partial y \ll \partial V_{wx}/\partial h$), the time rate of change of the wind velocity can be simplified to the following.

$$\dot{V}_{wx}(t, x, y, h) = \left(\frac{\partial V_{wx}}{\partial h} \right) \dot{h} = - \left(\frac{\partial V_{wx}}{\partial h} \right) V \sin \gamma \quad (33)$$

Substituting equation (33) and a similar expression for the wind derivative in the $\hat{\mathbf{n}}_2$ direction into the dynamics equations in (28)-(30) results in the following.

$$\dot{V} = \frac{T - D}{m} + g \sin \gamma + V \left(\frac{\partial V_{wx}}{\partial h} \cdot \cos \psi + \frac{\partial V_{wy}}{\partial h} \cdot \sin \psi \right) \sin \gamma \cos \gamma \quad (34)$$

$$\dot{\gamma} = \frac{-L \cos \phi + mg \cos \gamma}{mV} - \left(\frac{\partial V_{wx}}{\partial h} \cdot \cos \psi + \frac{\partial V_{wy}}{\partial h} \cdot \sin \psi \right) \sin^2 \gamma \quad (35)$$

$$\dot{\psi} = -\frac{L \sin \phi}{mV \cos \gamma} - \left(\frac{\partial V_{wx}}{\partial h} \cdot \sin \psi - \frac{\partial V_{wy}}{\partial h} \cdot \cos \psi \right) \tan \gamma \quad (36)$$

Equations (34)-(36) in combination with the kinematic equations for \dot{x} , \dot{y} , and \dot{h} , which are determined from the $\hat{\mathbf{n}}_1$, $\hat{\mathbf{n}}_2$, and $\hat{\mathbf{n}}_3$ components in equation (26), respectively, form the EOM for the point-mass aircraft model with non-constant winds.

3.2 Sign Convention for the Wind Gradients

Equation (33) shows how the time derivative of the wind velocity is determined using the wind gradient with respect to altitude, $\partial V_w/\partial h$, and the vertical rate, \dot{h} . If $dV_w/dt < 0$, the wind magnitude is decreasing with time; and, if $dV_w/dt > 0$, the wind magnitude is increasing with time.

An example wind profile is shown in Figure 3. Because the \hat{n}_3 direction has been defined as positive in the upward direction, the wind gradient is positive; i.e., $\partial V_w/\partial h > 0$.

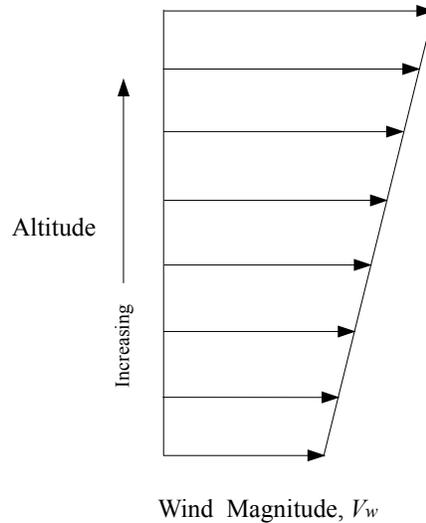


Figure 3. Example wind profile.

Given the wind profile in Figure 3, and if the aircraft is descending, $\partial V_w/\partial h > 0$ and $\dot{h} < 0$; therefore, $dV_w/dt < 0$ and the wind magnitude is decreasing with time, which matches the sign convention described above. If the aircraft is climbing, $\partial V_w/\partial h > 0$ and $\dot{h} > 0$; therefore, $dV_w/dt > 0$ and the wind magnitude is increasing with time.

4 Guidance Control Laws

The control inputs to the point-mass aircraft model are T , γ , and ϕ . Equation (29) is replaced with a first-order differential equation to track a commanded flight-path angle, γ_c , with gain k_γ ; wind-gradient effects on the flight-path angle tracking are still included.

$$\dot{\gamma} = k_\gamma (\gamma_c - \gamma) - \left(\frac{\partial V_{wx}}{\partial h} \cdot \cos \psi + \frac{\partial V_{wy}}{\partial h} \cdot \sin \psi \right) \sin^2 \gamma \quad (37)$$

First-order differential equations for thrust and roll angle also augment the equations of motion to model non-instantaneous changes to T and ϕ , where the gains k_T and k_ϕ are chosen to realistically model the aircraft's response.

$$\dot{T} = k_T (T_c - T) \quad (38)$$

$$\dot{\phi} = k_\phi (\phi_c - \phi) \quad (39)$$

Here, T_c and ϕ_c are the commanded thrust and roll angle, respectively.

The guidance control laws are designed to track a reference horizontal path, a reference calibrated airspeed (CAS), and a reference vertical profile. These reference inputs (and other inputs such as vertical rate) are determined from a pre-calculated four-dimensional trajectory that is generated by the aircraft's Flight Management System and is designed for a specified navigation procedure, which may include altitude and speed constraints at waypoints along the procedure.

When the aircraft is in level flight, the airspeed is controlled using thrust. However, there are two speed-management modes during the descent:

- Speed control using thrust: speed is managed using thrust and the altitude is managed using pitch (or flight-path angle in the point-mass model described here); and,
- Speed control using pitch: speed is managed by changing the pitch of the aircraft and the altitude is managed by modulating the thrust.

A speed brake model is also included to increase drag in cases when the aircraft needs to decelerate (in the speed control using thrust mode) or decrease its altitude (in the speed control using pitch mode), but is limited by the current state of the aircraft and/or its control inputs. When the speed brake is fully deployed, the drag coefficient is increased by 60%. To model the non-instantaneous deployment of the speed brake, the speed brake deployment fraction, b , is modeled as a first-order differential equation as shown.

$$\dot{b} = k_{speed\ brake} (b_c - b) \quad (40)$$

Here, $k_{speed\ brake}$ prescribes the rate of deployment and b_c is the commanded speed brake deployment fraction (e.g., 1.0 indicates full deployment and 0.5 indicates that the speed brake is only deployed halfway).

The lateral control law and the control laws for airspeed and altitude for each of the speed-management modes are described in the following subsections. Logic for deploying the speed brakes is included in the descriptions of the speed-management modes.

4.1 Lateral Control

The lateral control law is a function of the heading error, e_ψ , and the cross-track error, e_{xtrk} .

The heading error is the error in the commanded heading and the current heading.

$$e_\psi = \psi_c - \psi \quad (41)$$

The commanded heading is the sum of the course-angle guidance, θ , defined by the reference horizontal path, and the angle β , which is the angle between the horizontal projection of the TAS vector and the course angle.

$$\psi_c = \theta + \beta \quad (42)$$

The angle β is determined from the winds at the aircraft's current x, y position, the horizontal projection of the TAS, and the ground speed as illustrated in Figure 4.

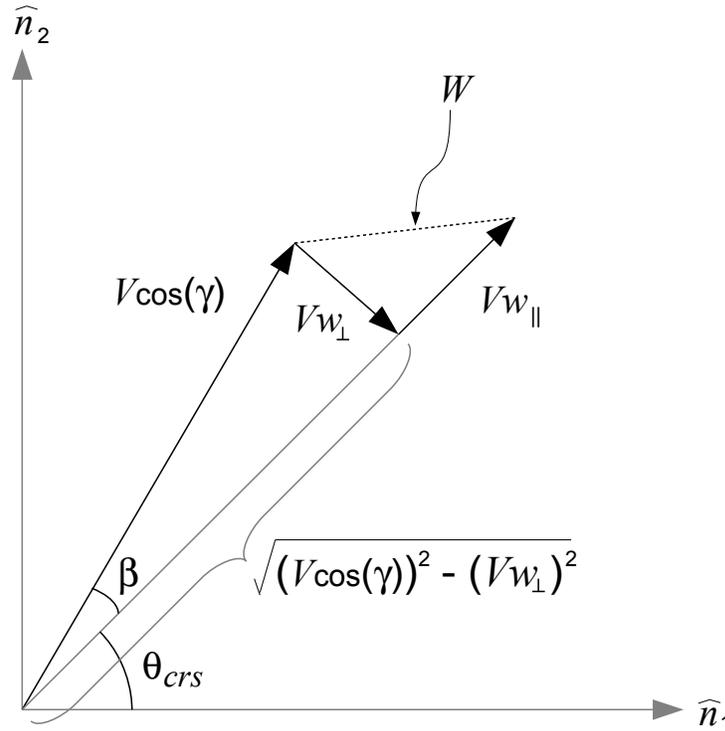


Figure 4. TAS and wind vectors in the horizontal plane.

The wind components parallel and perpendicular to the reference horizontal path, $V_{w\parallel}$ and $V_{w\perp}$, respectively, are found by rotating the x and y components of the wind through angle θ .

$$V_{w\parallel} = V_{wx} \cos \theta + V_{wy} \sin \theta \quad (43)$$

$$V_{w\perp} = -V_{wx} \sin \theta + V_{wy} \cos \theta \quad (44)$$

The ground speed, V_{gs} , and the wind magnitude, W , are calculated as shown.

$$V_{gs} = \sqrt{(V \cos \gamma)^2 - V_{w\perp}^2} + V_{w\parallel} \quad (45)$$

$$W = \sqrt{V_{wx}^2 + V_{wy}^2} \quad (46)$$

The magnitude of β is found using the Law of Cosines.

$$|\beta| = \cos^{-1} \left[\frac{(V \cos \gamma)^2 + V_{gs}^2 - W^2}{2V \cos \gamma \cdot V_{gs}} \right] \quad (47)$$

The sign of β is opposite to the sign of $V_{w\perp}$.

$$\beta = -|\beta| \cdot \text{sign}(V_{w\perp}) \quad (48)$$

The heading error should be limited to $[-\pi, \pi)$ to prevent a turn that is greater than 180 degrees.

The cross-track error is the perpendicular distance between the aircraft's current position and the reference horizontal path. Equations to calculate the cross-track error based on the aircraft's x , y position and the reference horizontal path are presented in Section 5.2.1.

The commanded roll angle is computed as a linear function of the heading and cross-track errors.

$$\phi_c = -k_\psi e_\psi - k_{xtrk} e_{xtrk} \quad (49)$$

4.2 Speed Control Using Thrust

The airspeed control law is a simple proportional control law that determines the desired acceleration from which the commanded thrust may be computed using dynamic inversion. The commanded calibrated airspeed, V_{CAS_c} , is converted to a commanded TAS, V_c . The desired acceleration, \dot{V}_c , is a function of the error between the current and the commanded TAS.

$$\dot{V}_c = k_V (V_c - V) \quad (50)$$

The commanded thrust, T_c , is found by solving equation (34) for T and substituting the desired acceleration for \dot{V} .

$$T_c = m\dot{V}_c + D - mg \sin \gamma - mV \left(\frac{\partial V_{wx}}{\partial h} \cdot \cos \psi + \frac{\partial V_{wy}}{\partial h} \cdot \sin \psi \right) \sin \gamma \cos \gamma \quad (51)$$

The commanded thrust is bounded by the maximum and minimum thrust values, T_{max} and T_{min} , respectively. Section 6 describes how T_{max} and T_{min} are calculated for a specific aircraft type.

The altitude control law commands a flight-path angle, γ_c , to manage the aircraft's altitude relative to the reference vertical profile defined by the reference altitude, h_{ref} , and the reference vertical rate, \dot{h}_{ref} .

$$\gamma_c = \sin^{-1} \left[\frac{-(\dot{h}_{ref} + k_{alt} e_{alt})}{V} \right], \text{ where } e_{alt} = h_{ref} - h \quad (52)$$

If the commanded thrust, T_c , is less than T_{min} , the current CAS is checked against the upper CAS limit for the next flaps configuration to determine if the flight crew can deploy the next flaps setting to increase drag.¹ The speed brake is deployed if the minimum thrust is commanded for more than 15 seconds and if the current TAS is more than 5 kt faster than V_c . The speed brake is deployed for at least 30 seconds or until the commanded thrust is greater than the minimum thrust (i.e., $T_c > T_{min}$).

The following algorithm describes the logic for checking the aircraft configuration in the case that $T_c < T_{min}$. The function *getConfigForDrag* is detailed in Section 6. At the start of the simulation, *minThrustCounter* is initialized to 0, and the current flaps configuration is initialized based on the aircraft's current CAS per the function *getConfig*, which is also detailed in Section 6.

Algorithm 1 Configuration for Increased Drag (Speed on Thrust)

```

if( $T_c < T_{min}$ ) then
     $T_c = T_{min}$ 
     $newFlapConfig = getConfigForDrag(V_{CAS}, h, h_{FAF}, currentFlapConfig)$ 
     $minThrustCounter = minThrustCounter + 1$ 
else
     $minThrustCounter = 0$ 
    if( $T_c > T_{max}$ )
         $T_c = T_{max}$ 
    end if
end if

```

The use of speed brakes is detailed in the following algorithms: Algorithm 2 is used to determine when to deploy speed brakes, and Algorithm 3 is used to determine when to retract speed brakes.

Algorithm 2 Speed Brake Deployment Logic (Speed on Thrust)

```

if( $minThrustCounter > 15$  AND  $(V_c - V) < -5.0$  kt) then
    if( $newFlapConfig == \{CR, AP, LDG\}$ ) then
        if( $speedBrakeOn == false$ ) then
             $speedBrakeCounter = 0$ 
             $b_c = 0$ 
             $speedBrakeOn = true$ 
        else
             $speedBrakeCounter = speedBrakeCounter + 1$ 
             $b_c = 0.5$ 
        end if
    else
         $speedBrakeCounter = 0$ 
         $b_c = 0$ 
         $speedBrakeOn = false$ 
    end if
end if

```

¹The aircraft model is assumed to have four potential flaps configurations: cruise (CR), approach (AP), landing (LDG), and landing with gear (LDG+GEAR), which are described more fully in Section 6.

Algorithm 3 Speed Brake Retraction Logic

```
if(speedBrakeOn == true AND speedBrakeCounter ≤ 30) then  
    speedBrakeCounter = speedBrakeCounter + 1  
     $b_c = 0$   
else if(speedBrakeOn == true AND speedBrakeCounter > 30)  
    if(minThrustCounter==0) then  
        speedBrakeCounter = 0  
         $b_c = 0$   
        speedBrakeOn = false  
    else  
        speedBrakeCounter = speedBrakeCounter + 1  
         $b_c = 0.5$   
    end if  
end if
```

4.3 Speed Control Using Pitch

This speed management mode is only used when the aircraft is in descent; otherwise, the speed is controlled using thrust. Therefore, additional logic is needed to determine when the aircraft is descending and when the aircraft is in level flight.²

The logic for determining the speed management mode is based on the altitude error, $e_{alt} = h - h_{ref}$, and the reference vertical rate, \dot{h}_{ref} . If one of the following conditions applies, the aircraft is assumed to be in level flight, and speed is managed using thrust:

- $e_{alt} < h_{threshold}$;
- the aircraft's current altitude is 200 feet (ft) below the lower bound on the next downstream altitude constraint in the navigation procedure; or
- $\dot{h}_{ref} = 0$

Here, $h_{threshold}$ is a tolerance on the altitude error measured relative to the reference altitude. If the aircraft is in level flight, the commanded thrust is determined using equation (51) and $\gamma_c = 0$.

If the aircraft's current altitude is more than 200 ft above the lower bound on the next downstream altitude constraint, the aircraft is assumed to be descending, and speed is managed using pitch.

The commanded flight-path angle is computed based on the Energy Share Factor (ESF), which is the ratio of power allocated to changing altitude versus changing airspeed. The error in TAS is $e_V = V_c - V$, and ESF_0 is the Energy Share Factor needed to maintain the aircraft's current airspeed while descending. The algorithm outlines the logic for determining the ESF needed for accelerating or decelerating in order to achieve the commanded speed, V_c .

²This paper only deals with managing the aircraft during level or descending flight; i.e., climbing flight is not considered.

Algorithm 4 Logic for Determining Energy Share Factor (Speed on Pitch)

```

if( $e_V \leq -V_{threshold}$ ) then
   $ESF = 0.3$ 
else if( $e_V > -V_{threshold}$  AND  $e_V \leq 0$ ) then
   $ESF = \left(\frac{ESF_0-0.3}{V_{threshold}}\right)e_V + ESF_0$ 
else if( $e_V > 0$  AND  $e_V \leq V_{threshold}$ ) then
   $ESF = \left(\frac{1.7-ESF_0}{V_{threshold}}\right)e_V + ESF_0$ 
else if( $e_V \geq V_{threshold}$ )
   $ESF = 1.7$ 
end if

```

Here, $V_{threshold}$ is a parameter used to determine when the ESF should be set to the minimum or maximum values. If the aircraft needs to decelerate ($e_V < 0$), the ESF is decreased (towards a minimum value of 0.3), meaning that the vertical path should become shallower. If the aircraft needs to accelerate ($e_V > 0$), the ESF is increased (towards a maximum value of 1.7), meaning that the vertical path should become steeper. The ESF values used in this logic are proposed in EUROCONTROL's BADA user's manual [2].

The commanded flight-path angle is then calculated using the following equations.

$$\frac{dh}{dt} = \left(\frac{(T - D) \cdot V}{mg}\right) \cdot ESF \quad (53)$$

$$\gamma_c = \sin^{-1}\left(\frac{-\frac{dh}{dt}}{V}\right) \quad (54)$$

The commanded thrust is based on e_{alt} and is calculated using the following logic.

Algorithm 5 Calculation of Commanded Thrust (Speed on Pitch)

```

if( $e_{alt} < -h_{threshold}$ ) then
   $T_c = 0.5 \cdot T_{max}$ 
else if( $e_{alt} > h_{threshold}$ ) then
   $T_c = T_{min}$ 
else
   $T_c = \left(\frac{T_{min}-0.5 \cdot T_{max}}{2 \cdot h_{threshold}}\right)e_{alt} + \frac{T_{min}+0.5 \cdot T_{max}}{2}$ 
end if

```

Again, T_{min} is the idle-thrust setting and T_{max} is the maximum thrust. If the current altitude is below the reference altitude by more than $h_{threshold}$, the commanded thrust, T_c , is set to half of the maximum thrust, T_{max} . If the current altitude is above the reference altitude by more than $h_{threshold}$, the commanded thrust is set to the minimum thrust, T_{min} .

In this speed management mode, Algorithm 6 is used to determine whether the next flaps configuration or speed brakes should be deployed to increase drag. As mentioned previously, Algorithm 3 is used to determine when to retract speed brakes.

Algorithm 6 Configuration for Increased Drag (Speed on Pitch)

```

if ( $T_c == T_{min}$ ) then
  if ( $e_{alt} > h_{threshold}$ ) then
     $newFlapConfig = getConfigForDrag(V_{CAS}, h, h_{FAF}, currentFlapConfig)$ 
    if ( $newFlapConfig == currentFlapConfig$  AND  $newFlapConfig == \{CR, AP, LDG\}$ )
      then
         $speedBrakeCounter = speedBrakeCounter + 1$ 
         $b_c = 0.5$ 
         $speedBrakeOn = true$ 
      end if
    end if
  end if
   $minThrustCounter = minThrustCounter + 1$ 
else
   $minThrustCounter = 0$ 
end if

```

4.4 Control Gains and Parameter Values

The control gains and thresholds used in the guidance control laws are presented in Table 1.

Table 1. Control Gains and Parameter Values

Control Gain	Value
k_T	$0.352 s^{-1}$
k_ϕ	$0.4 s^{-1}$
$k_{speed\ brake}$	$0.10 s^{-1}$
k_ψ	3.0
k_{xtrk}	$5 \times 10^{-4} m^{-1}$
k_V	$0.1136 s^{-1}$
k_{alt}	$0.20 s^{-1}$
$h_{threshold}$	500 ft
$V_{threshold}$	10 kt

5 Reference Horizontal Path

The aircraft is controlled to a reference horizontal path, a reference altitude, and a reference CAS, which are determined from a four-dimensional trajectory. The reference altitude and airspeed are provided as a function of along-path position (or distance-to-go [DTG]); therefore, the reference inputs are determined using the aircraft's along-path position.

The reference horizontal path is described in detail in this section, including an algorithm for mapping the aircraft's x, y position to an along-path position.

5.1 Data Format for the Horizontal Path

The reference horizontal path is comprised of a series of straight and turning segments that, when put together, describe a continuous path for the aircraft. The data format is provided in Figure 5.

	A	B	C	D	E	F	G	H	I	J
HPT _j	x	y	DTG	Segment Type	Course	Turn Center x	Turn Center y	Angle Start of Turn	Angle End of Turn	R
1	0	0	0	straight	6.2814	0	0	0	0	0
2	5279.26	-9.23	5279.3	turn	1.00E+07	5285.72	3684.91	-1.5725	-1.0487	3694.14
3	7127.86	482.84	7214.3	straight	0.5221	0	0	0	0	0
4	10593.80	2476.82	11212.9	turn	1.00E+07	8007.20	6973.01	-1.0487	-0.6128	5187.14
5	12250.50	3989.59	13474.2	-	-	-	-	-	-	-

Figure 5. Data format for the reference horizontal path.

The horizontal path example in Figure 5 describes a reference path with two straight segments and two turning segments, which is illustrated in Figure 6. The reference horizontal path is generated backwards from the last point at $x = 0, y = 0$ (which is the first line in Figure 5). Each x, y position in Columns A and B represents a Horizontal Path Transition (HPT) point, where the horizontal path transitions between a straight segment and a turn segment and vice versa or connects two straight segments. The DTG to the end point from each HPT_j is given in Column C. The segment type (straight or turn) that follows HPT_j is given in Column D.³ If the segment between two transition points is straight, the course, defined on $[0, 2\pi)$, is provided in Column E⁴; if the segment is a turn, the course value is set to 1.0×10^7 . For turn segments, columns F, G, H, I, and J include the x and y coordinates of the turn center, the angles of the start and end of the turn, and the turn radius. The angles at the start and end of the turn are used to determine the sweep angle of the turn, are defined on $[-\pi, \pi)$, and are measured relative to the x -axis. All values in Figure 5 are given in meters or radians.

³It should be noted that when the aircraft is flying along the horizontal path, the segment in line j precedes HPT_j .

⁴The course value given in Figure 5 is the course used for generating the horizontal path backwards from the last point; i.e., the flown course is the value in Figure 5 + π .

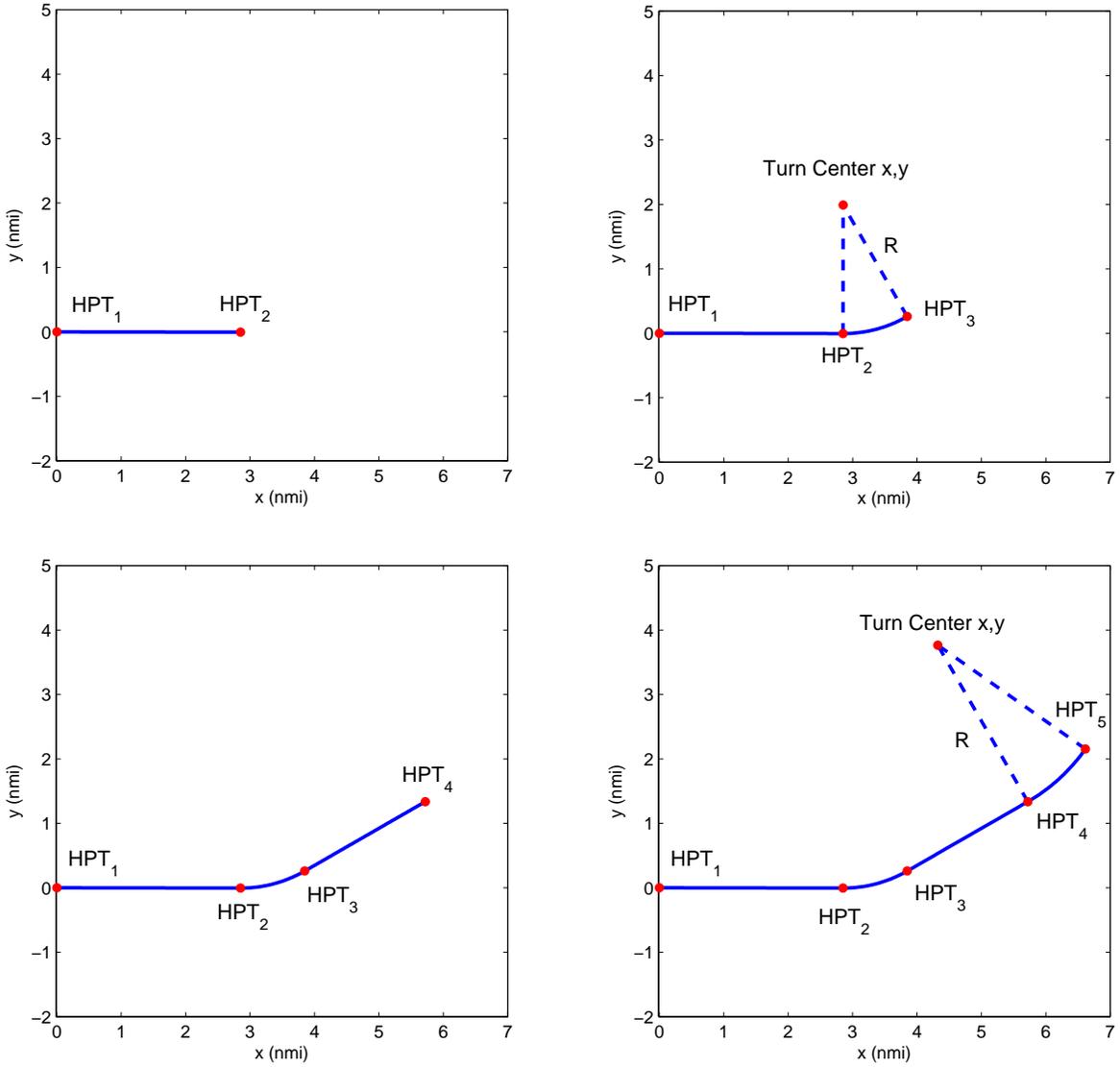


Figure 6. Horizontal path generated from the data in Figure 5. The horizontal path starts with the last point $(x, y) = (0,0)$ and the path is generated backwards using straight and turn segments, as specified. The first straight segment is shown in the top-left, the first turn segment is appended to first straight segment in the top-right, the second straight segment is added in bottom-left, and the second turn segment is appended in the bottom-right.

5.2 Mapping to Along-Path Position

The along-path position of the aircraft is found by orthogonally projecting the aircraft's x, y position onto the reference horizontal path. Figure 7 illustrates the high-level process for determining the along-path position.

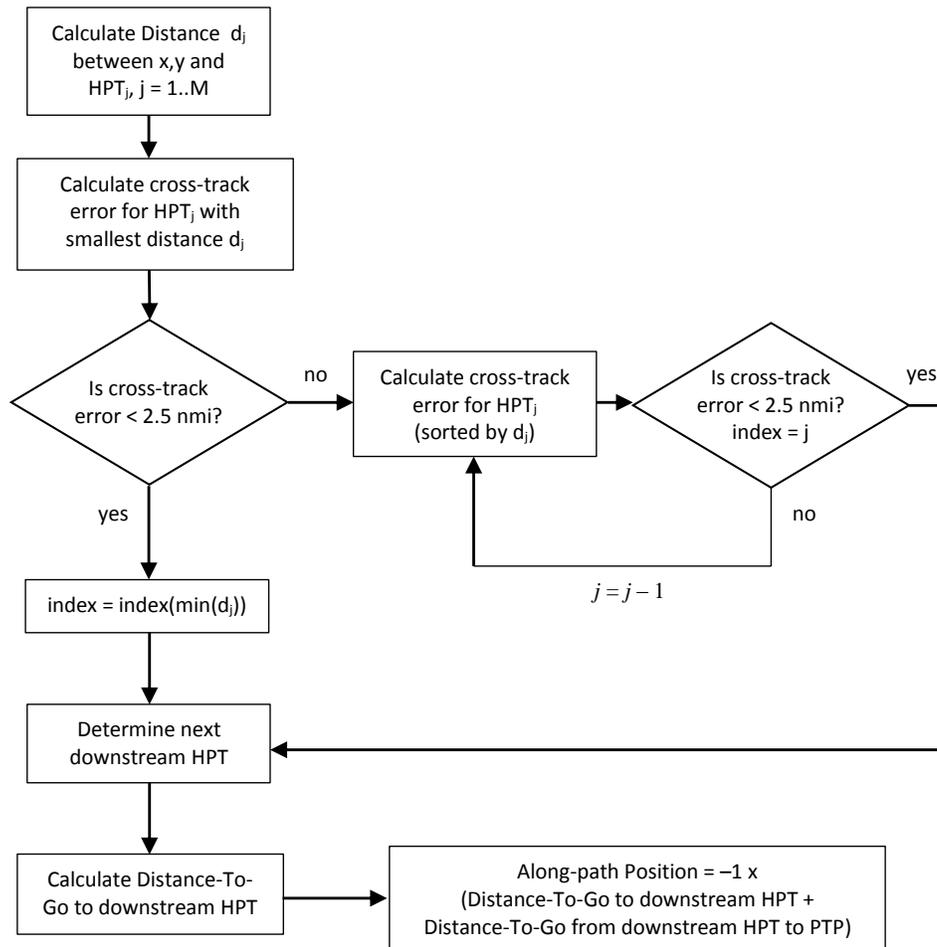


Figure 7. High-level process for determining the along-path position by projecting the aircraft's x, y position onto the reference horizontal path.

The distance between the x, y position and HPT_j for $j = 1..M$ is calculated using the Euclidean distance. Starting with the HPT point that results in the smallest distance, d_j , the cross-track error is calculated between the aircraft's position and the horizontal path.

If the absolute value of the cross-track error is less than 2.5 nmi, then determine the next downstream HPT. Otherwise, check the cross-track error for the HPT with the next smallest value of d_j . Continue checking the cross-track errors until finding an HPT that meets the 2.5-nmi threshold. The DTG and along-path position are then calculated using the next downstream HPT.

The cross-track error and along-path position calculations are detailed below.

5.2.1 Cross-track Error Calculation

The cross-track error is calculated by first identifying the whether the aircraft is on a straight or a turn segment. This is done by defining four equal quadrants around HPT_j and determining which quadrant the aircraft's x, y position falls within. Figure 8 shows a horizontal path defined by HPT_j for $j = 1..4$, comprised of two straight segments and one turn segment. If $j = 2$, meaning that the aircraft is closest in distance to HPT_2 , the quadrants are defined using $\alpha = \text{course}(HPT_2) + \pi$. If $j = 1$ or $j = 3$, then $\alpha = \text{course}(HPT_1) + \pi$ or $\alpha = \text{course}(HPT_3) + \pi$, respectively. The four quadrants are then defined as follows.

- quadrant 1: $[\alpha, \alpha + \pi/2)$
- quadrant 2: $[\alpha + \pi/2, \alpha + \pi)$
- quadrant 3: $[\alpha + \pi, \alpha + 3\pi/2)$
- quadrant 4: $[\alpha + 3\pi/2, \alpha + 2\pi)$

The angle between the aircraft's position and HPT_j is defined on $[0, 2\pi)$ and is determined as:

$$\theta_{HPT_j} = \text{atan2}(y - y(HPT_j), x - x(HPT_j)) \quad (55)$$

To determine the quadrant where the aircraft is located, θ_{HPT_j} is checked against the bounds of each quadrant. For example, if θ_{HPT_j} is within quadrant 1, then $\text{diff}_1 = |\alpha - \theta_{HPT_j}| \leq \pi/2$ AND $\text{diff}_2 = |\alpha + \pi/2 - \theta_{HPT_j}| \leq \pi/2$. It should be noted the differences in angles must be defined on $[-\pi, \pi)$.

The following logic is then applied to determine the next downstream HPT based on the quadrant that contains the aircraft's position and the segment type.

Algorithm 7 Logic for Determining Downstream HPT

```

if[segment( $HPT_j$ ) == turn AND (quadrant == 1 OR quadrant == 4)] then
    nextHPT =  $j - 1$ 
else if[segment( $HPT_j$ ) == turn AND (quadrant == 2 OR quadrant == 3)] then
    nextHPT =  $j$ 
else if[segment( $HPT_j$ ) == straight AND (quadrant == 1 OR quadrant == 4)] then
    if  $j == 1$  then
        nextHPT = 1
    else
        nextHPT =  $j - 1$ 
    end if
else if[segment( $HPT_j$ ) == straight AND (quadrant == 2 OR quadrant == 3)] then
    nextHPT =  $j$ 
end if

```

$nextHPT$ is then used to determine whether the aircraft is on a straight or a turn segment: $segment(nextHPT)$. Figure 8 shows three examples of aircraft positions in the horizontal plane: a , b , and c . If the aircraft is located at position a , $nextHPT = 1$ and the aircraft is assumed to be on a straight segment.

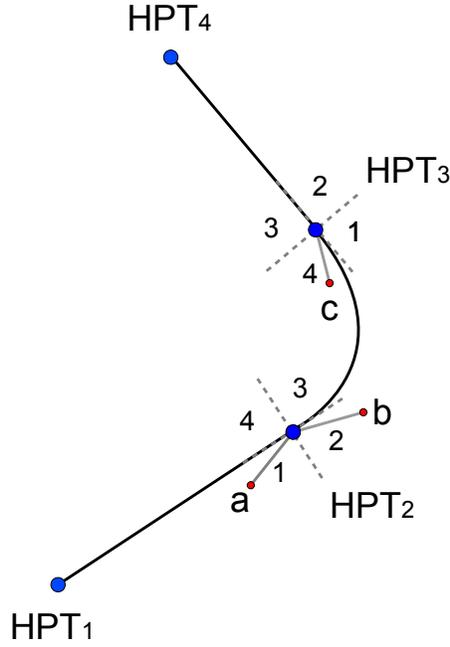


Figure 8. Illustration of quadrants around each HPT.

If the aircraft is located at positions b or c , $nextHPT = 2$ and the aircraft is assumed to be on a turn segment.

For straight segments, the cross-track error, e_{xtrk} , is calculated using the following equations.

$$\theta = \text{course}(nextHPT), \quad \theta : [0, 2\pi) \quad (56)$$

$$e_{xtrk} = -[x - x(nextHPT)] \sin \theta + [y - y(nextHPT)] \cos \theta \quad (57)$$

If the aircraft is determined to be on a turn segment, the cross-track error is found using the following equations.

$$R_1 = \sqrt{(x - x_{turn_center}(nextHPT))^2 + (y - y_{turn_center}(nextHPT))^2} \quad (58)$$

$$e_{xtrk} = |R_1 - R(nextHPT)| \cdot \theta_{dir} \quad (59)$$

Here, θ_{dir} is the direction of the turn. If the course angle at the end of the turn is greater than at the start of the turn, $\theta_{dir} = 1$; otherwise, $\theta_{dir} = -1$.

5.2.2 Along-Path Position Calculation

The DTG to the end point is calculated as a function of the downstream HPT point and the segment type (straight or turn).

The DTG is the cumulative path length from the next HPT to the end point, $L(nextHPT)$, plus the distance from the aircraft's current position to $nextHPT$, $DTG(nextHPT)$.

$$DTG = L(nextHPT) + DTG(nextHPT) \quad (60)$$

If the aircraft is on a straight segment, the $DTG(nextHPT)$ is given by the following.

$$DTG(nextHPT) = [x - x(nextHPT)] \cos \theta + [y - y(nextHPT)] \sin \theta \quad (61)$$

If the aircraft is on a turn segment, the following equations are used to find $DTG(nextHPT)$.

$$\theta_{turn} = \text{atan2}(y - y_{turn_center}(nextHPT), x - x_{turn_center}(nextHPT)) \quad (62)$$

$$\Delta\theta = \theta_{turn_start} - \theta_{turn}, \quad \Delta\theta : [-\pi, \pi] \quad (63)$$

$$DTG(nextHPT) = R \cdot |\Delta\theta| \quad (64)$$

Figure 9 illustrates the quantities used to calculate the DTG for both a straight and a turn segment. From aircraft position a , the DTG is calculated to the downstream point HPT_1 . From aircraft position c on the turn segment, the distance to HPT_2 is the arclength of the projection onto the horizontal path, and the DTG is the sum of that distance and the distance from HPT_2 to HPT_1 .

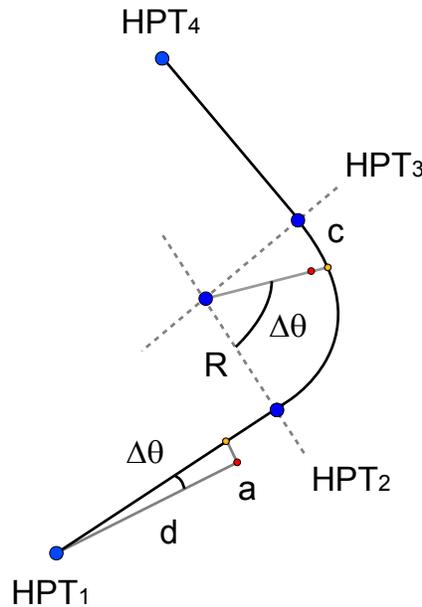


Figure 9. Projection of aircraft positions onto horizontal path to determine DTG to end point (HPT_1).

It is assumed that the along-path positions are negative and decreasing along the path to zero at the final point. Therefore, the DTG is multiplied by -1 to get the along-path position.

6 Aircraft-Specific Modeling

EUROCONTROL's Base of Aircraft Data (BADA) is used to represent aircraft-specific performance [2]. This section lists the BADA parameters and equations from the BADA user's manual that are used in the aircraft model and the control laws presented here.

The aircraft mass parameters are taken from the Operations Performance File (OPF) for the specified aircraft type. Three mass parameters are needed:

- Reference Mass, m_{ref}
- Minimum Mass, m_{min}
- Maximum Mass, m_{max}

The aircraft mass should be bounded by the minimum and maximum mass values.

The wing reference area, S , which is used in the computation of the lift coefficient, is also given in the OPF.

The drag parameters, C_{D0} and C_{D2} , are given in the OPF by configuration. For modeling descent trajectories, the cruise ($C_{D0,CR}$, $C_{D2,CR}$), approach ($C_{D0,AP}$, $C_{D2,AP}$), and landing ($C_{D0,LDG}$, $C_{D2,LDG}$) configurations are used. The landing gear configuration is modeled by adding the drag coefficient for the landing gear, $C_{D0,\Delta LDG}$, to the drag coefficients for the landing configuration. The drag coefficients, C_D , in the cruise, approach, and landing configurations are given by equations (BADA:3.6-2)-(BADA3.6-4).

The minimum transition speeds for each configuration are described in BADA: Section 3.5 (b) as a function of the stall speed for each configuration (corrected using the mass of the aircraft and assuming a stall speed margin). The transition (CAS) speed limits for each configuration are denoted as $V_{AP,trans}$, $V_{LDG,trans}$, and $V_{LDG+GEAR,trans}$. The following function, *getConfig*, is applied to determine whether the aircraft should transition to next flaps configuration based on the aircraft's current CAS and the minimum speed for its current configuration.

Algorithm 8 Get Configuration Based on Transition Speeds

```

if(currentFlapConfig == {CR} AND  $V_{CAS} > V_{AP,trans}$ ) then
    newFlapConfig = CR
else if(currentFlapConfig == {CR, AP} AND
 $V_{CAS} \leq V_{AP,trans}$  AND  $V_{CAS} > V_{LDG,trans}$  AND  $(h - h_{FAF}) < 10,000$  ft) then
    newFlapConfig = AP
else if(currentFlapConfig == {CR, AP, LDG} AND
 $V_{CAS} \leq V_{LDG,trans}$  AND  $V_{CAS} > V_{LDG+GEAR,trans}$  AND  $(h - h_{FAF}) < 10,000$  ft) then
    newFlapConfig = LDG
else if(currentFlapConfig == {CR, AP, LDG, LDG + GEAR} AND
 $V_{CAS} < V_{LDG+GEAR,trans}$  AND  $(h - h_{FAF}) < 10,000$  ft) then
    newFlapConfig = LDG + GEAR
else

```

newFlapConfig = *currentFlapConfig*
end if

Because the commanded CAS, V_{CAS} , must be limited to the upper CAS speed limit for a given configuration, the upper speed limits for each configuration ($V_{AP,max}$, $V_{LDG,max}$, and $V_{LDG+GEAR,max}$) will also be provided for each aircraft type. The function *getConfigForDrag*, described below, is used to determine whether the next flaps configuration can be deployed to increase drag. Inputs to the function are the aircraft's current CAS, V_{CAS} , the aircraft's current altitude, h , and the current configuration, *currentFlapConfig*.

Algorithm 9 Get Configuration for Increased Drag

if(*currentFlapConfig* == *CR* AND $V_{CAS} < V_{AP,max}$ AND $(h - h_{FAF}) < 10,000$ ft) **then**
 newFlapConfig = *AP*
else if(*currentFlapConfig* == *AP* AND $V_{CAS} < V_{LDG,max}$ AND $(h - h_{FAF}) < 10,000$ ft) **then**
 newFlapConfig = *LDG*
else if(*currentFlapConfig* == *LDG* AND $V_{CAS} < V_{LDG+GEAR,max}$ AND $(h - h_{FAF}) < 10,000$ ft)
then
 newFlapConfig = *LDG + GEAR*
else
 newFlapConfig = *currentFlapConfig*
end if

The drag force, D , is calculated as a function of air density, wing reference area, and the TAS using equation (BADA:3.6-5). The lift force is calculated similarly.

$$L = \frac{C_L \cdot \rho \cdot V^2 \cdot S}{2}$$

The thrust parameters are also given in the OPF and are as follows:

- $C_{Tc,1}$
- $C_{Tc,2}$
- $C_{Tc,3}$
- $C_{Tdes,high}$
- $C_{Tdes,low}$
- $C_{Tdes,app}$
- $C_{Tdes,ld}$
- $H_{p,des}$

The maximum engine thrust, T_{max} , is modeled using equations (BADA:3.7-1) and (BADA:3.7-8), assuming no temperature deviations from the standard atmospheric model. The minimum

thrust, T_{min} , is modeled as a function of altitude and configuration using equations (BADA:3.7-9)-(BADA:3.7-12).

The maximum operating calibrated airspeed (CAS) and Mach are given in the OPF as V_{MO} and M_{MO} , respectively.

The standard atmospheric model is given in Section BADA:3.1; conversions between CAS and TAS are described in equations (BADA:3.1-23) and (BADA:3.1-24).

7 References

- [1] J. E. Hurtado, *Kinematic and Kinetic Principles*. Hurtado - Lulu.com, 2007.
- [2] “User manual for the base of aircraft data (BADA) revision 3.11,” Tech. Rep., May 2013, Eurocontrol Experimental Center Technical/Scientific Report No. 13/04/16-01.

Appendix A Abbreviations and Acronyms

3-DOF	Three Degrees of Freedom
4D	Four Dimensional
<i>AP</i>	Approach Flaps Configuration
<i>b</i>	Speed brake deployment fraction
<i>b_c</i>	Commanded speed brake deployment fraction
BADA	Base of Aircraft Data
CAASD	Center for Advanced Aviation System Development
CAS	Calibrated Airspeed
<i>CR</i>	Cruise Flaps Configuration
DTG	Distance to Go
<i>D</i>	Drag
e.g.	For Example
EUROCONTROL	European Organization for the Safety of Air Navigation
FAA	Federal Aviation Administration
FAF	Final Approach Fix
ft	Feet
<i>g</i>	Acceleration due to gravity
γ	Flight-path angle
γ_c	Commanded flight-path angle
<i>h</i>	Altitude of the aircraft (+ <i>h</i> is pointing up from the surface of the Earth)
<i>h_{ref}</i>	Reference altitude from pre-calculated trajectory
\dot{h}_{ref}	Reference vertical rate from pre-calculated trajectory
HPT	Horizontal Path Transition
i.e.	That Is
IM	Interval Management
<i>k_{alt}</i>	Altitude tracking gain
<i>k_φ</i>	Roll-angle tracking gain
<i>k_ψ</i>	Heading-angle tracking gain
<i>k_{speedbrake}</i>	Speed-brake-deployment tracking gain
<i>k_T</i>	Thrust tracking gain
<i>k_v</i>	True airspeed tracking gain
<i>k_{xtrk}</i>	Cross-track error gain
kt	Knots

L	Lift force
LDG	Landing Flaps Configuration
$LDG + GEAR$	Landing Flaps Configuration plus Landing Gear
m	Aircraft mass
nmi	Nautical Miles
ϕ	Roll angle
ϕ_c	Commanded roll angle
ψ	Heading angle
ψ_c	Commanded heading angle
sec	Seconds
T	Thrust force
T_c	Commanded thrust force
θ	Course-angle guidance from the reference horizontal path
$\theta_{nextHPT}$	Angle between the x, y position of the aircraft and the next downstream Horizontal Path Transition
θ_{turn}	Angle between the x, y position of the aircraft and the center of the turn for the next downstream Horizontal Path Transition
TOD	Top of Descent
V	True airspeed of the aircraft
\dot{V}	Time rate of change of the TAS
\mathbf{V}	Inertial velocity vector
$V_{AP,max}$	Maximum CAS in the Approach Flaps Configuration
$V_{AP,trans}$	Minimum Speed to transition from Cruise Flaps Configuration to the Approach Flaps Configuration
V_c	Commanded TAS
V_{CAS_c}	Commanded calibrated airspeed
$V_{LDG,max}$	Maximum CAS in the Landing Flaps Configuration
$V_{LDG,trans}$	Minimum Speed to transition from Approach Flaps Configuration to the Landing Flaps Configuration
$V_{LDG+GEAR,max}$	Maximum CAS in the Landing Flaps + Gear Configuration
$V_{LDG+GEAR,trans}$	Minimum Speed to transition from Landing Flaps Configuration to the Landing Flaps + Gear Configuration
\mathbf{V}_w	Wind velocity vector
$V_{w\parallel}$	Wind speed aligned with aircraft's course
$V_{w\perp}$	Wind speed perpendicular to aircraft's course
V_{w_x}	Wind speed in the x direction
V_{w_y}	Wind speed in the y direction
x	Position of the aircraft in the x direction in the horizontal plane ($+x$ direction is aligned with East)

y

Position of the aircraft in the y direction in the horizontal plane (+ y direction is aligned with North)

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